Rancho Bernardo High School/Math Department
Honors Pre-Calculus Exit Exam

You are about to take an exam that will test your knowledge of the RBHS Honors Pre-Calculus curriculum. You must demonstrate genuine understanding of key concepts and mastery of problem solving skills in order to be prepared for AP Calculus AB/BC.

Please read all directions carefully. Errors made due to not following directions will result in loss of points. Unless otherwise stated in the directions, your answers should be stated as **exact** values. This means no decimal approximations.

You are permitted to use **a Scientific Calculator only** with no graphing capability.

This test should be completed in approximately three hours.

You will score your own test and tally up your points. Based on your score you can determine your preparedness level for AP Calculus AB/BC. There are a total of 300 points possible on the test.

Score

0 - 200	not recommended to proceed into AP Calculus AB/BC
201 - 300	recommended to proceed into AP Calculus AB/BC

RBHS Honors Pre-Calculus Exit Practice Exam

Instructions: Show ***all relevant work.*** Your work should be neat, organized and easy to follow. Simplify all answers to lowest terms. Give ***exact*** answers. ***Only a Scientific Calculator is permitted!!***

1. State the domain of each of the following: *(3 pts. each)*

a.) $f(x)=\dfrac{x^5}{|2x|-6}$ **b.)** $g(x)=\log\left(9x-x^2\right)$ **c.)** $h(x)=x-4\ln\left(-\dfrac{1}{x}\right)$

d.) $i(x)=Sin^{-1}x$ **e.)** $j(x)=\dfrac{1}{\sqrt{|x^2-1|}}$ **f.)** $g(x)=\dfrac{4x}{\sqrt{16-x^2}}$

2. Determine which of the following functions are odd, even, or neither. *(2 pts. each)*

a.) $f(x)=\dfrac{x^5}{|2x|-6}$ **b.)** $g(x)=\sqrt{9-x^2}$ **c.)** $h(x)=\dfrac{x^2-5x^3}{3}$

d.) $i(x)=Tan^{-1}x$ **e.)** $j(x)=x\sin x$ **f.)** $g(x)=\dfrac{4x}{\sqrt{16-x^2}}$

3. Given: $P(x)=2x^5-5x^3+3x^2$ and $Q(x)=7x^2+3x+2$

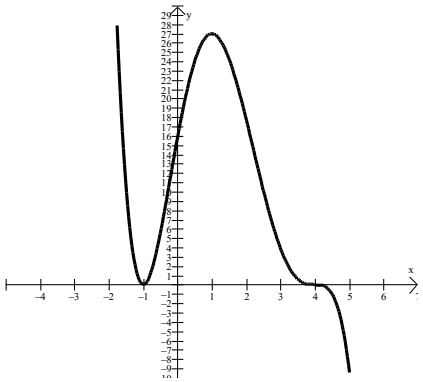
a.) List all possible rational roots of $P(x)=0$ _____ *(2 pts)*

b.) Write $P(x)$ in completely factored form: _____ *(6 pts)*

c.) Find values of *x* for which $P(x)=Q(x)$ _____ *(6 pts)*

d.) Find values of *x* for which $P(x)<Q(x)$ _____ *(3 pts)*

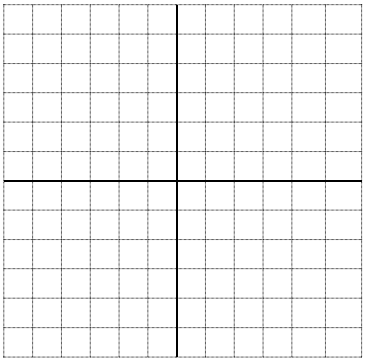
4. Write the polynomial function for the given graph. Find the exact equation containing the point (1, 27). Leave in factored form. *(5 pts)*



5. Find the value of the constant *k* so that $P(x) = x^3 - kx^2 + 4x - 8$ has $-2i$ as one of its roots. *(5 pts)*

#6 - 7: Provide a reasonable sketch of each function. State the domain, range, and zeros of the function (give exact values).

6. $f(x) = x^3 + 3x^2$



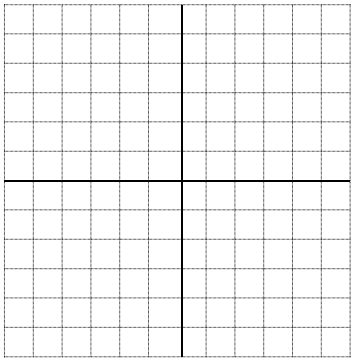
(3 pts)

Domain _____ *(2 pts)*

Range _____ *(2 pts)*

Zeros _____ *(2 pts)*

7. $f(x) = -2 + \sqrt{36 - x^2}$



(3 pts)

Domain _____ *(2 pts)*

Range _____ *(2 pts)*

Zeros _____ *(2 pts)*

8. Water is leaking out of a conical tank at a rate of 4π ft^3/min. If the height is 12 feet and the base radius is 4 feet,

a.) Find the total volume of the tank. *(2 pts)*

b.) Express the volume of the tank as a function of its height, *h*. *(3 pts)*

c.) How long will it take to completely empty the tank? *(2 pts)*

#9 – 14: Solve the following equations over the Real numbers. Give exact answers. *(5 pts. each)*

9. $e^{2x} - 2e^{x} = 0$

10. $3x - |2x-1| = 2x+1$

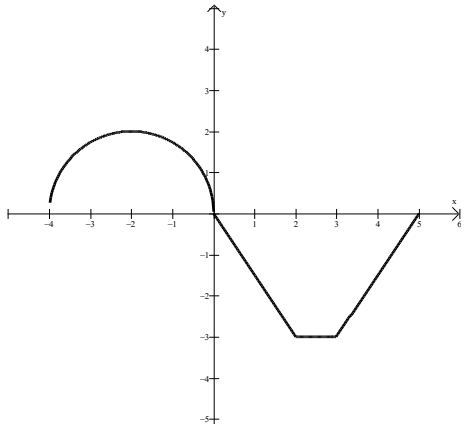
11. $4^{2x^2} = \left(\frac{1}{2}\right)^{-3x-1}$

12. $\log_4(4x) - \log_4(2+x) = \frac{1}{2}$

13. $4^{x} - 2^{x} - 12 = 0$

14. $2x^{\frac{7}{5}} - 8x^{\frac{2}{5}} = 0$

15. The following graph shows one full period of $y = f(x)$, a periodic function. The function is defined for all real numbers.



a.) State the period of $f(x)$. _____ *(1 pt)*

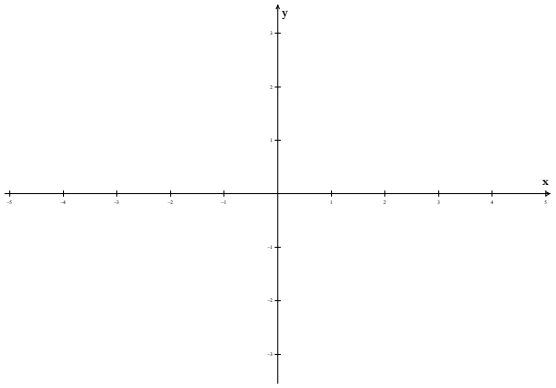
b.) State the amplitude of $f(x)$. _____ *(2 pts)*

c.) Find $f(1002)$. _____ *(2 pts)*

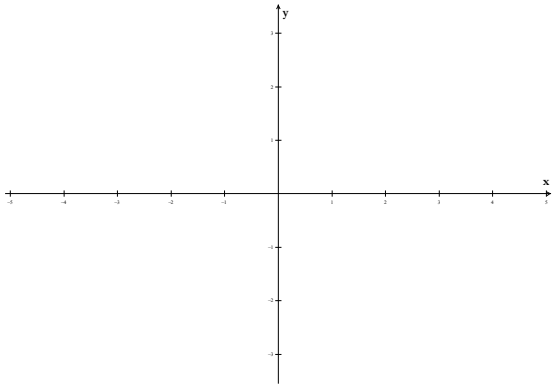
d.) Find $f(-82)$. _____ *(2 pts)*

#16 – 19: Sketch the graph of each transformation of the function shown above to the left . *(4 pts. each)*

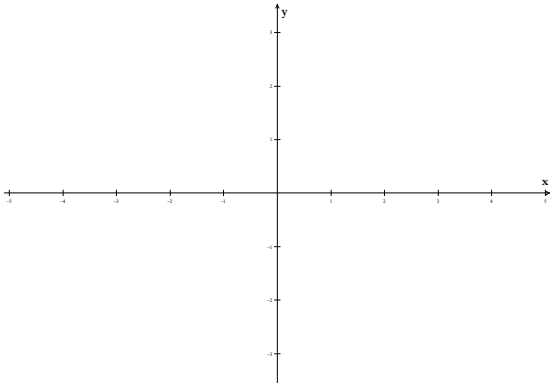
16. $y = f\left(-\frac{1}{2}x\right)$



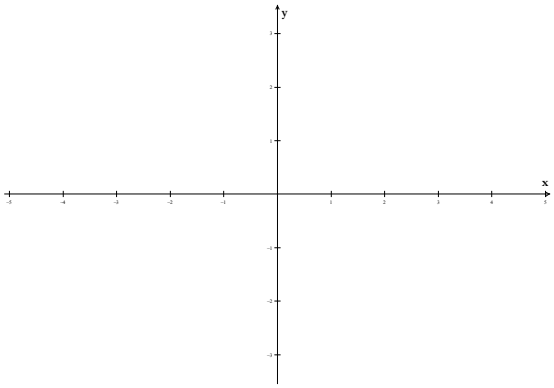
17. $y = |f(x) - 2|$



18. $y = -f(-x)$



19. $y = f(|x|)$



20. Write the function $g(x) = |x| - |x-4| - 2x$ in "piecewise" form. *(6 pts.)*

#21: Solve each for $0 \le x < 2\pi$ **.** *(6 pts. each)*

21. a.) $2\cot^2 3x + 1 + \csc 3x = 0$

b.) $\sin 2x \cos x = \sin x$

22. For what values of *x*, for $0 \le x < 2\pi$ **, is** $\cos 2x < \sin x$ **?** *(6 pts.)*

#23– 26: Given $\frac{\pi}{2} < \beta < \pi < \alpha < \frac{3\pi}{2}$ **, where** $\sin\beta = \frac{\sqrt{14}}{4}$ **and** $\cot\alpha = \frac{4}{3}$ **, find the EXACT value of each:** *(5 pts each)*

23. $\sin(\alpha - \beta)$

24. $\tan 2\alpha$

25. $\cos(\pi + \alpha)$

26. $\sin\left(\frac{\pi}{2} - \beta\right)$

#27 – 28: Solve for <u>all</u θ **in radians. Give exact answers.** *(5 pts each)*

27. $-6\tan\left(\theta + \frac{3\pi}{4}\right) - 2 = 4$

28. $\frac{\csc\frac{\pi}{3}\theta}{8} = \frac{1}{4}$

#29 –30: Simplify. Give exact answers. *(4 pts each)*

29. $\frac{1}{2}\sin^2\frac{5\pi}{12}-\frac{1}{2}\cos^2\frac{5\pi}{12}$

30. $\sin\left[\frac{4\pi}{3}+Tan^{-1}\left(\frac{12}{5}\right)\right]$

31. Find the angle of elevation θ for the line $2x-4y=7$ in degrees to the nearest tenth. *(4 pts)*

32. State the domain and range of each of the following: *(1 pt each)*

a.) $y=-3\sin(x-\pi)-4$

Domain: _____

Range: _____

b.) $y=\tan 3x$

Domain: _____

Range: _____

c.) $y=2\csc\frac{\pi}{3}x$

Domain: _____

Range: _____

d.) $y=Cos^{-1}x$

Domain: _____

Range: _____

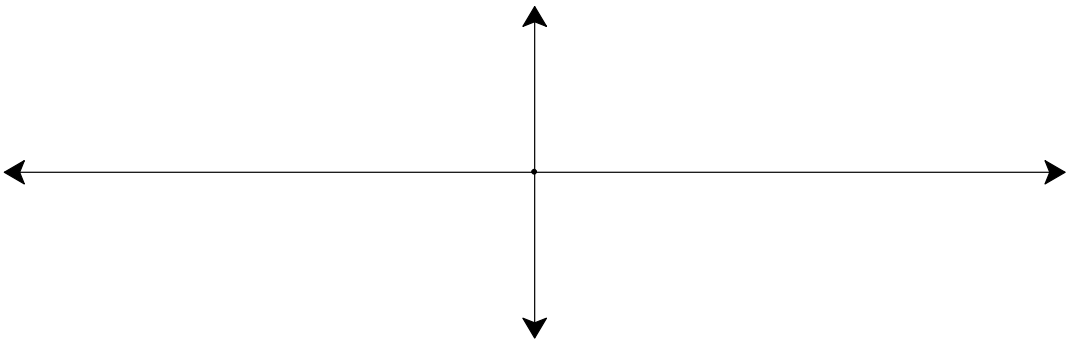
33. Given ΔCAT, $m\angle T = 20°$, $c = 6$ cm, and $t = 4$ cm. Solve the triangle. Find angles and side lengths to the nearest tenth.

(7 pts)

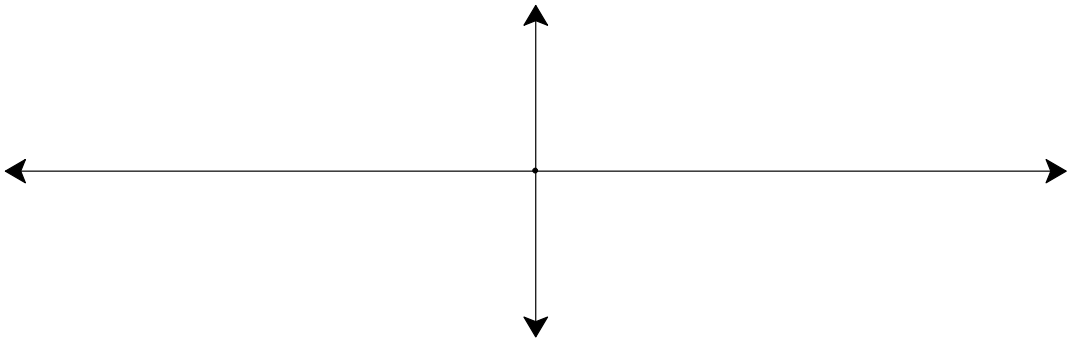
34. Three ships are assigned to rescue an astronaut as his spaceship plunges into the ocean. The ships are at the vertices of a triangle with side lengths 5, 7, and 10 kilometers. Find the measure of the largest angle of this triangle to the nearest tenth of a degree. *(5 pts)*

35. For each of the following, graph over the indicated domain. List and label all pertinent information. *(6 pts each)*

a.) $y = -2\cot\frac{1}{2}x$ for $-2\pi < x < 2\pi$.

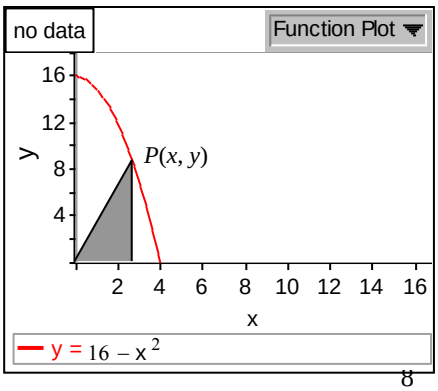


b.) $y = -3\cos 2x$ for $-2\pi \le x < 2\pi$.



36. A point $P(x, y)$ lies in the first quadrant of the parabola $y = 16 - x^2$ as shown in the diagram.

a.) Express the area of the shaded triangular region as a function of the x coordinate of P. *(3 pts)*



b.) What is the domain of the function? *(2 pts)*

37. Given the series: $\frac{2^2}{49} - \frac{9^4}{64} + \frac{16^6}{81} - \frac{23^8}{100} + \ldots - \frac{(163)^{48}}{900}$

a.) Write the series in summation notation using $k=0$ as a lower limit. *(4 pts)*

b.) Write the series in summation notation using $k=3$ as a lower limit. *(4 pts)*

38. Given the infinite geometric series: $f(x)=\frac{(x-2)}{2}+\frac{(x-2)^2}{6}+\frac{(x-2)^3}{18}\ldots$

a.) Find the interval of convergence of the series. *(4 pts)*

b.) Solve for all values of *x* if the sum of the series converges to $x-2$. *(4 pts)*

c.) Find $f(1)$ if it exists. *(2 pts)*

d.) Find $f(-1)$ if it exists. *(2 pts)*

39. A rectangular flat piece of cardboard is going to be used to make a rectangular open top box by cutting out squares of equal size, each of which has side of length *x*, from each corner. If the dimensions of the cardboard are 14 inches by 8 inches, set up the equation for the volume of the box in terms of length *x* that is cut out. State the domain for volume of the box. *(5 pts)*

40. Find $f^{-1}(x)$ for each of the following. State the domain of $f^{-1}(x)$. *(4 pts each)*

a.) $f(x)=1-\sqrt[3]{2x-5}$

b.) $f(x)=\dfrac{1-x}{2x}$

c.) $f(x)=2-\sqrt{x+3}$

d.) $f(x)=2\ln(x-3)$

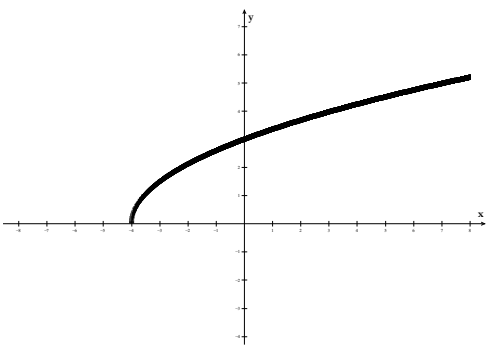
41. Answer the following given the function $f(x)=x^5+4x^3-3$. *(3 pts each)*

a.) If $f^{-1}(2x)=-1$, find the value of *x*.

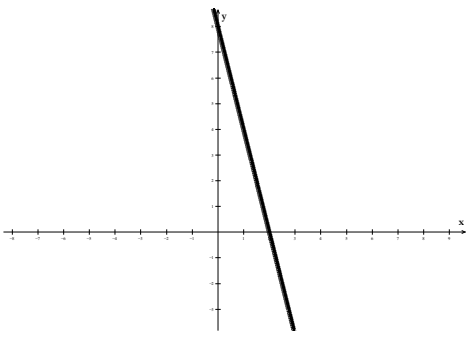
b.) If $f(x)=-8$, find the value of *x*.

42. Sketch the graph of $f^{-1}(x)$ given the graph of $f(x)$. *(2 pts each)*

a.)



b.)



x	$f(x)$
1	6
2	5
3	-1
4	-3

43. Find each of the following from the chart shown above given that both the function f and its inverse exist for all x. *(1 pt each)*

a.) $f^{-1}(-1)$

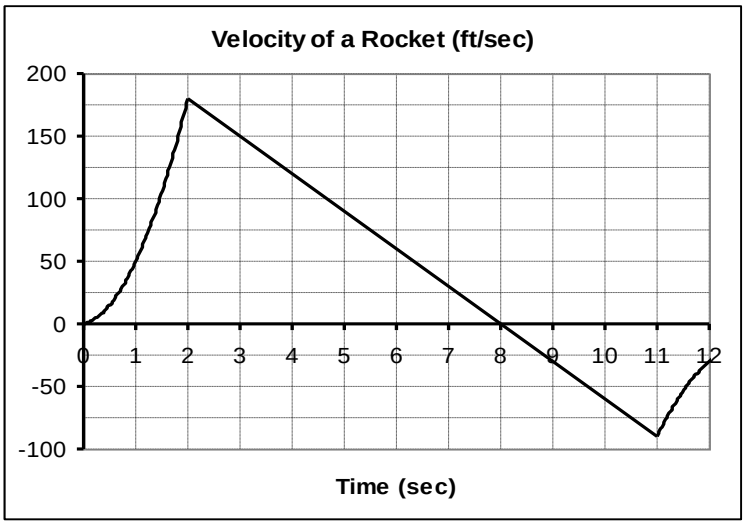
b.) Find x if $f(x)=5$.

c.) Find x if $f\left(-\frac{x}{2}\right)=-3$

d.) $f\left(2f^{-1}(6)\right)$

e.) $f^{-1}\left(-\frac{1}{2}f^{-1}(5)\right)$

f.) $f^{-1}(4x-2)=1$



44. A model rocket, launched from the ground level, burns fuel for a few seconds, accelerating the rocket upward. After the rocket engine burns out, the rocket coasts upward for a while and then begins to fall. A parachute pops out shortly after the rocket starts down in order to slow the rocket. Use the graph above to answer the questions *a-f* below. *(2 pts each)*

a.) At what time(s), in seconds, is the rocket's velocity 50 ft/sec?

b.) At what time, in seconds, did the rocket's engine burn out?

c.) At what time, in seconds, did the rocket reach its' highest point?

d.) At what time, in seconds, did the rocket's parachute pop out?

e.) What does the area under the curve represent in context to the rocket's flight? Recall $d = rt$.

f.) Approximate the maximum height of the rocket?

Answers:

1. a.) $x \neq \pm 3$ b.) $x \in (0,9)$

   c.) $x < 0$ d.) *All Real Numbers*

   e.) $x \neq \pm 1$ f.) $x \in (-4,4)$

2. a.) odd b.) even

   c.) neither d.) odd

   e.) even f.) odd

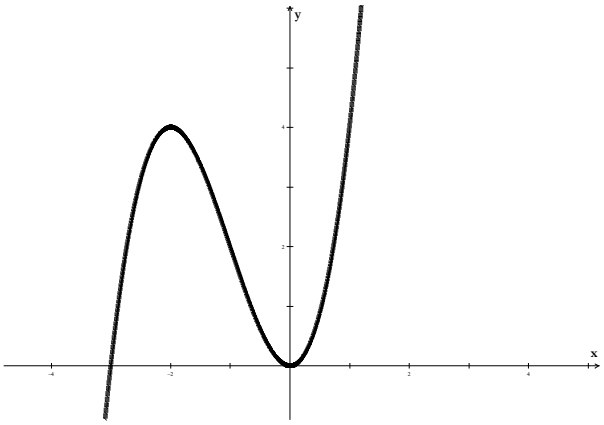
3. a.) $0, \pm 1, \pm \frac{1}{2}, \pm \frac{3}{2}$ b.) $x^2(x-1)(2x^2+2x-3)$

   c.) $x = -1, 2$ d.) $x \in (-\infty, -1) \cup (-1, 2)$

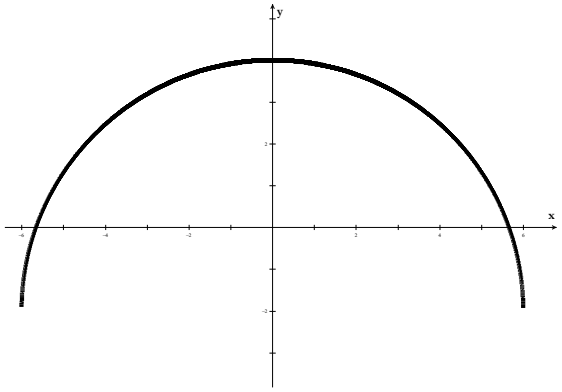
4. $-\frac{1}{4}(x-4)^3(x+1)^2$

5. k = 2; roots = 2i, -2i, 2

6. Domain: $x \in (-\infty, \infty)$
   Range: $y \in (-\infty, \infty)$
   Zeros: $x = -3, 0$

   6. 

7. Domain: $x \in [-6, 6]$
   Range: $y \in [-2, 4]$
   Zeros: $x = \pm 4\sqrt{2}$

   7. 

8 a.) $V = 64\pi$ cubic feet

   b.) $V = \frac{8}{3}\pi h$

   c.) 16 minutes

9. $\ln 2$ 10. $\varnothing$ 11. $x = 1, -\frac{1}{4}$

12. $x = 2$ 13. $x = 2$ 14. $x = 0, 4$

15. a.) 9 c.) -3

b.) $\frac{5}{2}$ d.) $\sqrt{3}$

16. reflects across *y*-axis with double the period

17. shifts down 2 then reflects across *x*-axis

18. reflect across the *x*-axis then the *y*-axis (symmetry about the origin)

19. everything to the left of the *y*-axis is ignored and everything to the right of the *y*-axis remains the same and then that is reflected across the *y*-axis (even)

20. $g(x)=\begin{cases}-2x-4, & x<0\\ -4, & 0\le x<4\\ -2x+4, & x\ge 4\end{cases}$

21. a.) $\left\{\frac{\pi}{2}\right\}$ b.) $\left\{0,\frac{\pi}{4},\frac{3\pi}{4},\pi,\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

22. $\frac{\pi}{6}<x<\frac{5\pi}{6},\ \frac{5\pi}{6}<x<\frac{3\pi}{2}$

23. $\frac{3\sqrt{2}-4\sqrt{14}}{20}$ 24. $\frac{24}{7}$

25. $-\frac{4}{5}$ 26. $\frac{\sqrt{2}}{4}$

27. $\theta=\pi k,\ k\in\mathbb{Z}$ 28. $\theta=\frac{1}{2}+6k,\ k\in\mathbb{Z}$ $\theta=\frac{5}{2}+6k,\ k\in\mathbb{Z}$

29. $\frac{\sqrt{3}}{4}$ 30. $\frac{-5\sqrt{3}-12}{26}$

31. $\theta=26.6^{\circ}$

32. a.) Domain: *All Real Numbers* b.) Domain: $x\ne\frac{\pi}{6}+\frac{\pi}{3}k,\ k\in\mathbb{Z}$

Range: *All Real Numbers* Range: $-7\le y\le -1$

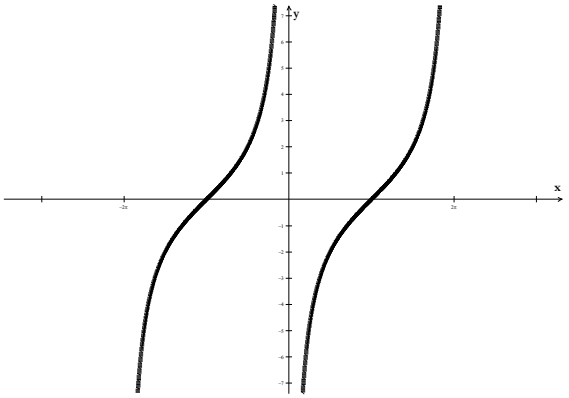
c.) Domain: $x\ne 3k,\ k\in\mathbb{Z}$ d.) Domain: $-1\le x\le 1$

Range: $y\le -2,\ y\ge 2$ Range: $0\le y\le \pi$

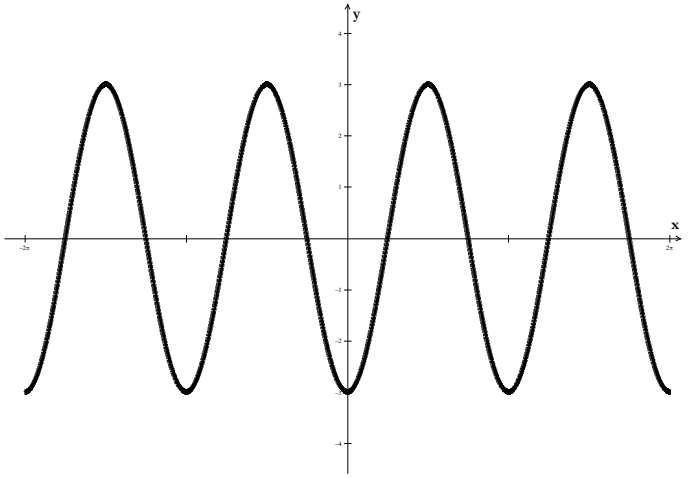
33. $m\angle A = 129.1^\circ,\ m\angle C = 30.9^\circ,\ a \approx 9.1$

34. 111.8°

35. a.)



b.)



36. a.) $A = \frac{1}{2}x(16 - x^2)$ b.) $0 \le x \le 4$

37. a.) $\sum_{k=0}^{23} \frac{(-1)^n (7n+2)^{2n+2}}{(n+7)^2}$ b.) $\sum_{k=3}^{26} \frac{(-1)^{n+1} (7n-19)^{2n-4}}{(n+4)^2}$

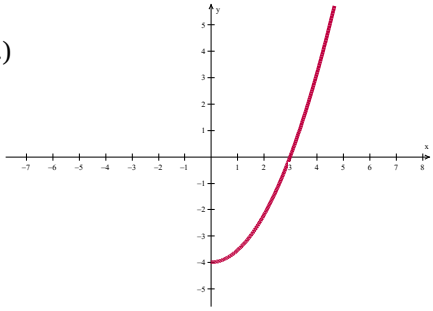
38. a.) $-1 < x < 5$ b.) $x = \frac{7}{2}, 2$ c.) $-\frac{3}{8}$ d.) does not exist

39.) $v(x) = x(14-2x)(8-2x)$ *domain* $0 \le x \le 4$

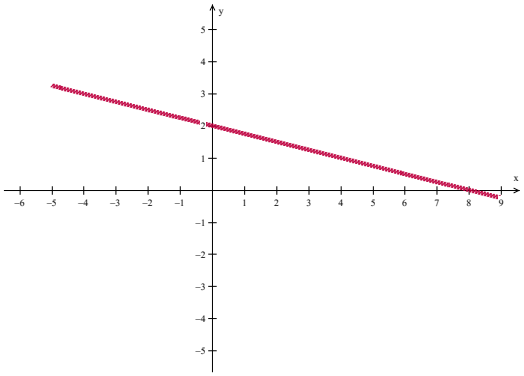
40. a.) $\frac{(1-x)^3 + 5}{2}$ b.) $\frac{1}{2x+1}$ c.) $(x-2)^2 - 3,\ x \le 2$ d.) $e^{\frac{x}{2}} + 3$

41. a.) $x = -4$ b.) $x = -1$

42. a.)



b.)



43. a.) 3 b.) 2 c.) -8 d.) 5 e.) 3 f.) 2

44. a.) 1, 6.2 sec b.) 2 sec c.) 8 sec d.) 11 sec e.) distance traveled by the rocket f.) about 650 feet