

K – 12 MATHEMATICS UNIFYING STANDARDS

- 1.0 Number Sense and Operations** – Students understand ways of representing numbers, relationships among numbers, and number systems. They understand the meaning of and relationships between operations and strategies, and they can estimate appropriately.
- 2.0 Patterns, Functions, and Algebra** – Students know and understand various types of patterns and functional relationships. They use symbolic forms and models to represent and analyze mathematical structures in both real and abstract contexts.
- 3.0 Measurement** – Students know and understand attributes, units and systems of measurement. They apply a variety of techniques, tools, and formulas for determining measurements.
- 4.0 Geometry and Spatial Sense** – Students know how to analyze characteristics and properties of two- and three- dimensional objects. They select and use different representational systems, including coordinate and graph theory. They understand the usefulness of transformations and symmetry in analyzing mathematical situations. They know how to visualize and to use spatial reasoning to solve problems that cross disciplines.
- 5.0 Data Analysis, Statistics, and Probability** – Students know how to pose questions and collect, organize, represent and interpret data in order to answer those questions. They use methods of exploratory data analysis to develop and evaluate inferences, predictions, and arguments that are based on data. They understand and know how to apply the notions of chance and probability.
- 6.0 Problem Solving** – Students know that they learn basic skills and concepts in order to use them to solve problems in and out of school. They solve routine and complex problems by drawing from a variety of strategies, including technology, while demonstrating an attitude of persistence and reflection in their approaches.
- 7.0 Processes: Reasoning, Communication, and Connections** – Students use reasoning to develop, analyze, draw conclusions, and validate conjectures and arguments. As they reason, they recognize and understand multiple representations of the same concept. They see the interconnections among math ideas, as well as in other disciplines. They know how to communicate their math thinking clearly and coherently to others, orally, graphically, and in writing, using precise language and symbols.

FOCUS GOALS CALCULUS

1.0 Number Sense and Operations

- 1.1 Understand and apply the concept of infinity and the density of the number line.
- 1.2 Understand and apply the concept of convergence and divergence in sequence and in series of numbers.

2.0 Patterns, Functions, and Algebra

- 2.1 Model and solve function problems using a graphing calculator.
- 2.2 Interpret patterns and relationships represented on graphs.
- 2.3 Analyze limits of functions.
- 2.4 Analyze asymptotic and unbounded behavior of functions.
- 2.5 Analyze the properties of continuity.
- 2.6 Compare relative magnitudes of functions and their relative rates of change.

3.0 Measurement

- 3.1 Utilize appropriate units of measurement when solving application related problems.
- 3.2 Apply Integrals to problems involving work, volume, velocity, and/or acceleration.

4.0 Geometry and Spatial Sense

- 4.1 Translate between the graphs of f , f' , and f'' and understand the geometric relationships between them.

5.0 Data Analysis, Statistics, and Probability

- 5.1 Solve and analyze differential equations that involve exponential growth/decay.

6.0 Problem Solving

- 6.1 Make decisions about how to approach problems.
- 6.2 Solve a variety of problems (real world and across disciplines) that require knowledge and application of derivatives.
- 6.3 Use strategies to apply the derivative graphically, functionally, and numerically to solve problems in physics, chemistry, economics, and so forth.
- 6.4 Solve a variety of problems that require knowledge and application of integrals.
- 6.5 Solve problems that require understanding and application of antidifferentiation techniques.
- 6.6 Apply definite integrals.
- 6.7 Use a calculator, computer, or both to approximate integrals numerically.

7.0 Processes: Reasoning, Communication, and Connections

- 7.1 Communicate math thinking and conclusions using words, graphs, and charts.
- 7.2 Apply literacy skills when making mathematical connections.

MATH STANDARDS CALCULUS

1.0 Number Sense and Operations – Students understand ways of representing numbers, relationships among numbers, and number systems. They understand the meaning of and relationships between operations and strategies, and they can estimate appropriately.

- 1.1 Understand and apply the concept of infinity and the density of the number line.** *
- Utilize the concept of infinity when finding the tangent to a curve or area under a curve.
- 1.2 Understand and apply the concept of convergence and divergence in sequence and in series of numbers.** (c/C23-24)

2.0 Patterns, Functions, and Algebra – Students know and understand various types of patterns and functional relationships. They use symbolic forms and models to represent and analyze mathematical structures in both real and abstract contexts.

- 2.1 Model and solve function problems using a graphing calculator.** (c/C)
- Apply Newton's method to approximate zeros. (c10)
 - Calculate the derivative of a function. (c4)
 - Apply the concept of continuity of a function. (c2)
 - Calculate the value of a definite integral. (c21)
 - Plot the graph of a function within an arbitrary viewing window. *
- 2.2 Interpret patterns and relationships represented on graphs.** (p)
- Identify interplay between geometric and analytical information.
 - Explain the observed local and global behavior of a function.
 - Predict local and global behavior of a function.
- 2.3 Analyze limits of functions.** (c/C1.1/p)
- Determine one-sided limits, limits at infinity, and infinite limits. (c1/p)
 - Calculate limits algebraically. (c1.3)
 - Estimate limits from graphs or tables of data. (p)
 - Use graphical calculators to verify and estimate limits. (c1/p)
 - Prove and use theorems evaluating the limits of sums, products, quotients, and composition of function. (c1.1)
- 2.4 Analyze asymptotic and unbounded behavior of functions.** (p/C/ma6)
- Explain asymptotes in terms of graphical behaviors. *
 - Describe asymptotic behavior in terms of algebraic limits involving infinity (horizontal, vertical, oblique). *

2.5 Analyze the properties of continuity. (c)

- Apply the intermediate value theorem. (c3)
- Apply the extreme value theorem. (c3)
- Understand and apply the formal definition and graphical interpretation of continuity of a function. (c2)
- Determine continuity of classes of functions (e.g., polynomial, trigonometric, logarithmic, rational, exponential, transcendental). *
- Provide proof of continuity of a point. *
- Calculate the value of a definite integral. (c8)

2.6 Compare relative magnitudes of functions and their relative rates of change. (c12)

- Compare and contrast exponential, polynomial, and logarithmic growth.

3.0 Measurement – Students know and understand attributes, units, and systems of measurement. They apply a variety of techniques, tools, and formulas for determining measurements.

3.1 Utilize appropriate units of measurement when solving application related problems. (p)**3.2 Apply Integrals to problems involving work, volume, velocity, and/or acceleration.** (c16)

- Calculate the length of a curve, the area of a surface revolution, volume of solids, and work. (c16)

4.0 Geometry and Spatial Sense – Students know how to analyze characteristics and properties of two- and three- dimensional objects. They select and use different representational systems, including coordinate and graph theory. They understand the usefulness of transformations and symmetry in analyzing mathematical situations. They know how to visualize and to use spatial reasoning to solve problems that cross disciplines.

4.1 Translate between the graphs of f , f' , and f'' and understand the geometric relationships between them. *

- Given the graph of a function, determine the properties of the function including maxima/minima, points of inflection, increasing/decreasing intervals. (c9)
- Given the graph of the derivative of a function, determine the properties of the function and sketch the function. (c9)
- Visual three-dimensional objects as a result of rotating a given region around a line. *

5.0 Data Analysis, Statistics, and Probability – Students know how to pose questions and collect, organize, represent, and interpret data in order to answer those questions. They use methods of exploratory data analysis to develop and evaluate inferences, predictions, and arguments that are based on data. They understand and know how to apply the notions of chance and probability.

5.1 Solve and analyze differential equations that involve exponential growth/decay.

(cb/p/c/C27)

- Apply appropriate calculus techniques to solve $y'=kx$ problems using separation of variables.
- Model data and make predictions. (s)

6.0 Problem Solving – Students know that they learn basic skills and concepts in order to use them to solve problems in and out of school. They solve routine and complex problems by drawing from a variety of strategies, including technology, while demonstrating an attitude of persistence and reflection in their approaches.

6.1 Make decisions about how to approach problems. (p9-12)

- Analyze problems by identifying relationships, discriminating relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
- Identify and utilize appropriate tools.
- Make generalizations based on prior knowledge and related examples.

6.2 Solve a variety of problems (real world and across disciplines) that require knowledge and application of derivatives. (cb/p/c/C4.2)

- Define and understand a derivative functionally and graphically. (cb/p/c4)
- Explain a derivative as an instantaneous rate of change (i.e., limit of secant lines). (c4.2)
- Recognize and understand the relationship between differentiability and continuity. (c6,4.3)
- Apply algorithmically, the derivative of algebraic, trigonometric, exponential, logarithmic, and other related inverse functions, including, but not limited to: (c4.4)
 - derivatives of higher order (c7.0)
 - logarithmic differentiation (c4.4)
 - chain rule (composite functions) (c5.0)
 - implicit differentiation (c6.0)

6.3 Use strategies to apply the derivative graphically, functionally, and numerically to solve problems in physics, chemistry, economics, and so forth. (cb/p/c4.2)

- Identify the slope of a curve, tangent, and normal lines. (c4)
- Identify intervals in which the function is increasing and decreasing. (c9)
- Use of tangent lines to approximate functional values. (c4.1)
- Apply critical numbers, points of inflection, and intervals of concavity. (cb/p)
- Sketch curve functions. (c9)

- Use differentiation to solve optimization problems (relative extrema – absolute). (c11)
- Apply rate of change (average/instantaneous). (c4.2)
- Apply rectilinear motion position, velocity, acceleration, and displacement versus total distance. (cb/p)
- Know and apply Rolle's Theorem and the Mean Value Theorem (interpretation and consequences). (c8.0)
- Differentiate the terms of a known power series to create a new one. (c25)

6.4 Solve a variety of problems that require knowledge and application of integrals.

(cb/c/C13/p)

- Use Riemann sums to define definite integrals and to approximate integrals. (c13)
- Apply the Fundamental Theorem of Calculus (and its proof) to interpret Integrals as antiderivatives. (c15/cb/p)
 1. If $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$
 2. $\int_a^b f(t)dt = F(b) - F(a)$ where $F'(x) = f(x)$
- Recognize and understand rectangular approximation with equal and unequal subdivisions. (cb/p)
- Recognize and understand trapezoid approximation with equal subdivisions (Trapezoid Rule). (cb/p)
- Recognize and apply the basic properties of definite Integrals. (cb/p)
 1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$
 2. $\int_a^b f(x)dx = 0$
 3. If $a < b < c$, then $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$
 4. If $K = \text{constant}$, then $\int_a^b Kdx = (b - a)K$
 5. If $f(x) = g(x)$, then $\int_a^b f(x)dx = \int_a^b g(x)dx$
 6. If $K = \text{constant}$, then $\int_a^b K(x)dx = K \int_a^b f(x)dx$

6.5 Solve problems that require understanding and application of antidifferentiation techniques. (c/C17/cb/p)

- Apply basic (non-composite) techniques: algebraic, trigonometric, exponential, and logarithmic functions. (c4.4/cb/p)
- Apply advanced techniques: substitution, long division, and integration by parts. (c17/cb/p)

6.6 Apply definite integrals. (cb/p)

- Determine area under and between curves.
- Determine average value of a function on an interval (MVT of Integral Calculus).

- Determine volume of solids with known cross sections and with washers/disks or cylindrical shells.
 - revolve around x-axis
 - y-axis
 - line parallel to either axis

6.7 Use a calculator, computer, or both to approximate integrals numerically. (c21)

- Understand Simpson's Rule and Newton's method. (c21)
- Computer integrals of trigonometric functions. (c20)
- Compute integrals of rational functions. (c19)
- Identify indefinite integrals as expressions of inverse trigonometric functions. (c18)

7.0 Processes: Reasoning, Communication, and Connections – Students use reasoning to develop, analyze, draw conclusions, and validate conjectures and arguments. As they reason, they recognize and understand multiple representations of the same concept. They see the interconnections among math ideas, as well as in other disciplines. They know how to communicate their math thinking clearly and coherently to others, orally, graphically, and in writing, using precise language and symbols.

7.1 Communicate math thinking and conclusions using words, graphs, and charts. (p)

7.2 Apply literacy skills when making mathematical connections. (p-language arts)

- Learn and use mathematics vocabulary encountered through reading. (R1.0)
- Use strategies to comprehend, analyze, and evaluate mathematics reading materials. (R2.0)
- Employ technology to organize and record mathematics information. (R3.1)
- Write about mathematics to convey ideas logically and correctly. (W2.4)